Exercise 15

The displacement (in meters) of a particle moving in a straight line is given by the equation of motion $s = 1/t^2$, where t is measured in seconds. Find the velocity of the particle at times $t = a$, $t = 1$, $t = 2$, and $t = 3$.

Solution

Determine the velocity first.

$$\begin{aligned}
v(t) &= s'(t) \\
&= \lim_{h\to 0} \frac{s(t+h) - s(t)}{h} \\
&= \lim_{h\to 0} \frac{\frac{1}{(t+h)^2} - \frac{1}{t^2}}{h} \\
&= \lim_{h\to 0} \frac{\frac{t^2}{t^2(t+h)^2} - \frac{(t+h)^2}{t^2(t+h)^2}}{h} \\
&= \lim_{h\to 0} \frac{\frac{t^2-(t+h)^2}{t^2(t+h)^2}}{h} \\
&= \lim_{h\to 0} \frac{t^2 - (t+h)^2}{ht^2(t+h)^2} \\
&= \lim_{h\to 0} \frac{t^2 - (t^2 + 2ht + h^2)}{ht^2(t+h)^2} \\
&= \lim_{h\to 0} \frac{-2ht - h^2}{ht^2(t+h)^2} \\
&= \lim_{h\to 0} \frac{-2t - h}{t^2(t+h)^2} \\
&= \frac{-2t}{t^2(t^2)} \\
&= -\frac{2}{t^3}
\end{aligned}$$

Therefore, the velocity at each of the times is

$$v(a) = -\frac{2}{(a)^3} = -\frac{2}{a^3}\ \frac{\text{m}}{\text{s}}$$

$$v(1) = -\frac{2}{(1)^3} = -2\ \frac{\text{m}}{\text{s}}$$

$$v(2) = -\frac{2}{(2)^3} = -\frac{1}{4}\ \frac{\text{m}}{\text{s}}$$

$$v(3) = -\frac{2}{(3)^3} = -\frac{2}{27}\ \frac{\text{m}}{\text{s}}.$$